

## Students' cognitive processes in understanding the application of derivatives

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**Abstract:** Derivatives are one of the objects of calculus that is difficult for students to learn. The purpose of this study was to describe the cognitive processes of students in understanding the application of derivatives. This is a qualitative research involving one subject with the initials L. The main instrument of this research is the researcher who is guided by an assignment sheet and an interview guide. Data collection was carried out through task-based interviews. To get complete and accurate data recorded through audiovisual. Data were analyzed descriptively through genetic decomposition based on APOS (action-process-object-schema) theory. The results of this study are that L can coordinate the process-object of all the properties of a given function, with adjacent or overlapping intervals in all  $h$  domains so that a mature scheme of function graph sketches is formed. So that an accurate function graph sketch is obtained. The conclusion is that L's cognitive process of applying derivatives is at a high level. It's a trance level.

**Kata Kunci:** Cognitive processes; application of derivatives; trans level; APOS

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### INTRODUCTION

Calculus is a subject that is difficult for students to understand. According to Dorier (2002), There are two main subjects that are difficult for science students to learn in the first two years, namely calculus and linear algebra. The basic concept of calculus that must be understood by students is limit and function derivatives. It is mandatory, because it will be used by students in understanding and solving further calculus problems. Tarmizi (2010) stated that there were student difficulties in solving calculus problems. Therefore, special treatment needs to be given to correct their misunderstanding. That is to avoid students getting confused in understanding calculus and solving problems in it.

In understanding calculus, there are many manifests. According to Tall (1993), Several types of calculus studied are informal calculus, formal analysis, infinitesimal ideas, and computer approaches. According to him, informal calculus is an informal notion of rates of change and rules of differentiation with integration as an inverse process, by calculating areas, volumes, etc. It is an integrated application. Formal analysis is the formal idea of completeness,  $\epsilon - \delta$  definition of limits, continuity, differentiation, Riemann integration, and formal deduction of theorems such as the average value theorem, the fundamental theorem of calculus, and the like. Also, with a variety of new approaches. Furthermore, a very small idea is based on non-standard analysis. Finally, a computer approach that uses one or more of the graphic manipulation facilities, numeric, symbolic, with or without programming. Calculus viewed from a variety of points of view can make us free to understand it, but it takes the teacher's understanding of the factors that affect student performance in understanding it.

According to Raj Acharya (2017), There are a number of interrelated factors that are detrimental to the underperformance of public school students in mathematics including calculus. It is the needs and interests of students with the level of knowledge and skills of

students in understanding mathematical concepts. Negative explanations about mathematics create frustration and anxiety in mathematics students. The teacher must eradicate the inhibiting factors for learning mathematics and make learning mathematics enjoyable in the context of students.

Based on another research perspective, that the types of difficulties faced by students in understanding calculus are students do not understand the concept of integral substitution, do not understand the concept of trigonometric comparisons, do not understand how to make examples of integral substitution and substitute into an integral form whose solution must be sought. Also, students do not understand about decomposition in trigonometry and do not understand what is decomposed and which is reduced, students do not know the formulas commonly used in trigonometry (A, Masriyah, & Manuharawati, 2019). In addition, students' difficulties in understanding calculus are drawing graphs of functions and performing trigonometric manipulations. Also, the students' difficulties regarding the function, namely determining the domain and range, mastery of the rules in determining the values of the boundary function. Students have difficulty regarding derivatives, namely determining the maximum and minimum scores in the story problem. The concept of integral is another difficulty that is the mistake of using the integral rule. It is mainly about applying derivatives, substitution integrals and partial integrals (Fatimah & Yerizon, 2019).

In one study, students were asked to sketch a graph of the function  $h$  which satisfies the following conditions:  $h$  is continuous;  $h(0) = 2$ ,  $h'(-2)=h'(3)=0$ , and  $\lim_{x \rightarrow 0} h'(x) = \infty$ ;  $h'(x) > 0$ , if  $-4 < x < -2$  and if  $-2 < x < 3$ ;  $h'(x) < 0$ , if  $x < -4$  and if  $x > 3$ ;  $h''(x) < 0$ , if  $x < -4$ , if  $-4 < x < -2$ , and if  $0 < x < 5$ ;  $h''(x) > 0$ , if  $-2 < x < 0$ , and if  $x > 5$ ;  $\lim_{x \rightarrow -\infty} h(x) = \infty$ ; and  $\lim_{x \rightarrow \infty} h(x) = -2$  (let's call it **Task 1**)(Baker, Cooley, Trigueros, & Trigueros, 2000). The results of his research found that there were students who could interpret a condition separately in relation to the graphical nature of a function. The student solely uses the first derived conditions even though he is aware of other properties but cannot coordinate these to produce a graph. If the two properties overlap, then students only use one trait to describe the graph. If a student tries to use more than one trait, then he or she cannot describe it completely and takes shortcuts using only one trait. It was found that other students started coordinating two or more conditions together. However, this coordination, has not been able to apply the overall condition. Also, there are students who can coordinate all the graph properties of a given function with all the intervals in the given function domain. Students demonstrate a function characteristic schema coherently.

A mature schema of a mathematical fragment is a coherent system of actions, processes, objects, and other schemes that have been built beforehand, which is coordinated and synthesized by individuals in the form of structures used to deal with certain problem situations. It is APOS Theory (action, process, object and schema)(Dubinsky, Dautermann, Leron, & Zazkis, 1994) (Dubinsky, 2000)(Dubinsky & McDonald, 2000). There are three levels of students' cognitive processes in understanding mathematics based on the APOS Theory. The three levels are intra, inter and the highest trans level (Dubinsky, 2010).

Students' cognitive levels are then developed into five levels, namely intra, semi-inter, inter, semi-trans, and trans levels which have herarchical and functional characteristics. (Widada, 2015). By utilizing genetic decomposition analysis (Widada, 2017), cognitive processes are developed into seven levels namely pre-intra, intra, semi-inter, inter, semi-trans, trans, and extended-trans levels (Widada, 2016)(Widada & Herawaty, 2017). Based on these explanations, we focus on this research on how students' cognitive processes in understanding the derivative application to sketch graphs of functions.

## METODE

This research is a qualitative study, which focuses on the task-based interview (Tsamir & Dreyfus, 2002)(Widada, Herawaty, Nugroho, & Anggoro, 2019b)(Widada, Nugroho, & Sari, 2018). This qualitative research as a procedure to reveal the nature of the symptoms that arise from the subject of research. The subjects of this study were 1 student who was selected based on the test to complete the task of sketching the function graph. The main instrument in this

study was the interviewer (in this case the researcher himself) and was guided by other instruments in the form of a task sheet on the problem of a graphical sketch of a non-routine function and a task sheet on the convergence of an infinite series, as well as an interview guide. The problem of graphical sketching of a nonroutine function referred to in this case is a question to sketch a graph of a function that is only given its analytical properties (first derivative, second derivative, limit, and continuity) (see Task 1 in the introduction). The selected subject (namely L) immediately carried out the data collection process through task-based interviews. The subject was responded to by a clinical interview in the form of a series of questions related to problem solving carried out by the subject, recorded via audiovisual. Data were analyzed by genetic decomposition analysis. Genetic decomposition analysis is an analysis of a structured collection of mental activities that build blocks (categories) to describe how concepts / principles can be developed in the mind of an individual. These mental constructions are actions, processes, objects, and schemes which form the theoretical framework of APOS Theory. This theory can be used directly in analyzing data about a person's schema behavior. Researchers can compare the success or failure of subjects to a math task through specific mental constructs that they may or may not perform. After completing the initial genetic decomposition, it is extensively analyzed again to revise the genetic decomposition that has been described previously to be more careful in reflecting the data.

## RESULTS AND DISCUSSION

The research subject (namely L) has carried out his duties. The results of the completion of the task regarding the h function graph sketch and the results of the task-based interview are presented below. Figure 1 is a Graphical Sketch of the function h made by L.

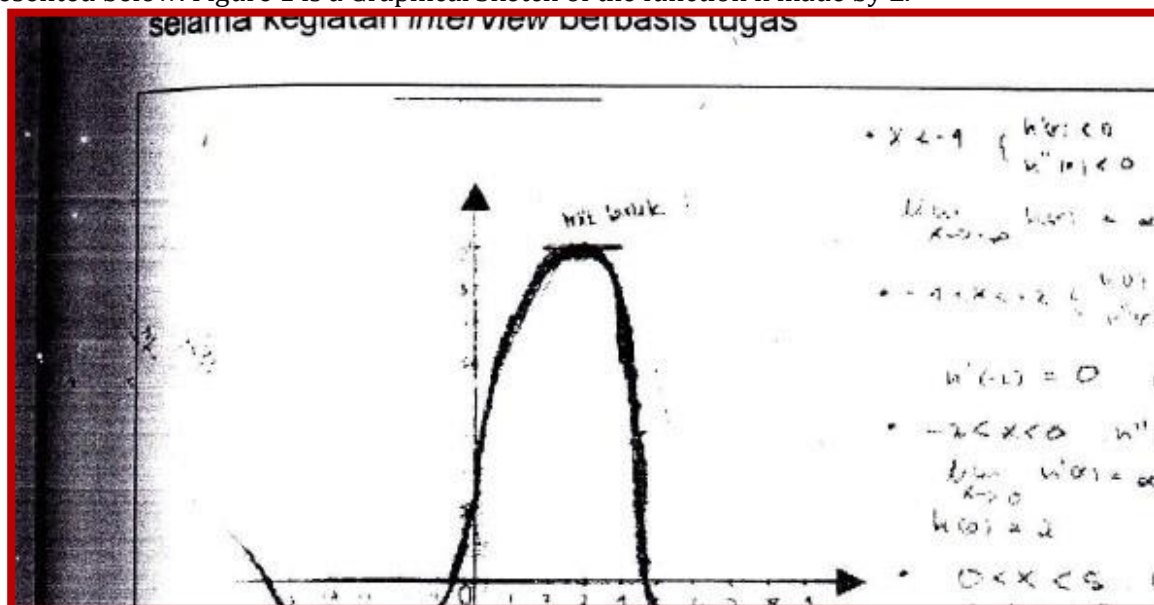


Figure 1. A graphic sketch made by L

Furthermore, based on genetic decomposition analysis of the h function graph sketch based on the task-based interview (between the Interviewer (P) and the research subject (L)). Analysis of genetic decomposition of L about the graph sketch of the function h. Conducted separately, namely the behavior of the schematic about the interval function domain h (symbolized A), and the schematic behavior regarding the nature of the function h (symbolized B), presented interview snippets and analysis sequentially.

### 1) Snippet 1

Q: Please read and understand the problem I gave, then you solve the problem.

L1.01: Yes! ... [.... Shut up....] [L nods, then reads and understands the problem for 5

minutes.]

P: Okay! Now please give an explanation of what you are doing!

L1.02: I will look at the interval first, where it goes up, it goes down, then where it is concave up, and concave down. [L while pointing at his work]

[Analysis: A: Schematic

L can coordinate cognitively adjacent or overlapping intervals in the  $h$  function domain into a mature scheme as described in L 1.02, and the arguments are in Figure 1.

B: Schematic

L can coordinate cognitively about all the properties of a given function at intervals in the  $h$  domain to form a mature scheme, as described in L 1.02, as well as the arguments in Figure 1.]

## 2) Snippet 2

L1.03: first of all for the interval  $-4 < x < -2$ ; There are two possibilities, namely  $h'(x) > 0$  which means the graph is rising to the right, and the fourth condition [fourth line] is  $h''(x) < 0$ , which means the graph is concave downward. [L sketches the graph as described L 1.03.]

P: Good!

L1.04: then I will see for the  $x < -4$  interval, there are also two conditions, namely  $h'(x) < 0$  and  $h''(x) < 0$ , meaning that the curve goes down to the right and is concave down [these things will mutually related], I drew this first, then later I combined it as a whole. [L sects the graph for the interval  $x < -4$  as expressed by L 1.04, as a continuation of L1.03, and he associates it with L 1.05.]

P: Okay! Very nice!

L1.05: In the  $x < -4$  interval there is also a limit of  $x$  leading to negative infinity equal to  $n$  infinity so that at the  $x < -4$  interval, (then the graph will continue to increase to the left). [L emphasizes the graph sketch of  $x < -4$  with L 1.05, meaning that the graph at the interval continues to increase to the left (down to the right), and concave down.]

P: Yach ...

L1.06: suppose for  $x = -4$  is the fracture point because for  $x < -4$  [the curve] is concave downward, [so is] for  $-4 < x < -2$ , and here it is continuous because it is known that  $h$  is continuous. [L denotes a break point as the argument L 1.06, and in  $-4$  a continuous sketched graph.]

[Analysis:

A: Object

L encapsulates processes about interval-intervals, adjacent or overlapping at  $x < -2$  into an object. This object is part of the mature schema in Snippet 1. Assuming the encapsulated process is a description of the intervals  $x < -4$ ,  $-4 < x < -2$  see L1.03-L 1.06. The formation of this scheme can also be seen from description L 1.04 that all of these descriptions will be related to being a whole.

B: Object

As the schema about internal develops, L also encapsulates the process of all the properties of the function  $h$  in all internal parts in the  $h$  domain, namely  $x < -2$ , so that an object is formed. This object is part of a mature scheme in Example 1. and the encapsulated processes are the description  $h'(x)$ , the limit of  $h(x)$  for  $x$  to negative takhin\_gga and  $h''(x)$  for  $x < -4$ , the description  $h'(x)$  and  $h''(x)$  for  $-4 < x < -2$ , and good coordination is done by L at  $x = -4$  so that L concludes that at  $x = -4$  is the fracture point, see L 1.03- L 1.06.1]

## 3) Snippet 3

L1.07: Next I will look at the interval  $-2 < x < 3$ , I took it first [that is],

$-2 < x < 0$  first, at this interval  $h''(x) > 0$  which means that it is concave upward, [and at that interval also the curve rises to the right because  $h'(x) > 0$ ], here is also a

condition that the limit of  $h'(x)$  for  $x$  approaches 0 is infinity, it means that there is a tangent which is almost parallel to the y-axis and almost coincides. And also here there is a condition  $h(0) = 2$ , meaning  $h$  through (0,2). Since the limit of  $h'(x)$  for  $x$  approaches 0 is infinity, so that the graph is almost closer [coincides] to the y-axis but does not coincide, the graph is steep. [L sketches the graph like the description of L1.07.]

P: Okay! Why turn at point-2?

L1.08: From -4 to -2 he [h curve] rises and is concave down and from -2 to 0, he is concave up and up, there it is up and concave down before -2 and concave up after -2 and before 0, so that there is a turn .. [L while sketching the graph of the function  $h$  corresponds to L 1.08.]

P: Why is -2 an extreme?

L1.09: [Because]  $h'(-2) = 0$  and according to the conditions  $h'(x) > 0$ , if  $-4 < x < -2$  and  $h''(x) > 0$ , if  $-2 < x < 0$ , so that di -2 is an inflection point. [L while sketching the graph of the function  $h$  corresponds to L 1.09.]

P: Okay!

L1.10: We continue, for intervals  $0 < x < 5$ ,  $h''(x) < 0$ , it means that [curve  $h$ ] is concave downward but between 0 and 5 there is an interval that we need to see, [ie]  $-2 < x < 3$ . Because  $-2 < x < 0$  is already us, we see  $0 < x < 3$ , where  $h'(x) > 0$  means rising to the right. After here it is almost steep he [curve  $h$ ] rises up to 3. Then for the second condition that for  $x > 3$ ,  $h'(x) < 0$ , this means he [curve  $h$ ] goes down to the right, at 3 it is said to be a batik point, this is concave down for  $0 < x < 3$ , this is also concave down to  $-3 < x < 5$ .

P why is it concave down?

L1.11: It is concave downwards, because from 0 to 5,  $h''(x) < 0$  .... so it is concave down.

P: Okay! Very nice!

L1.12: Then with the fourth condition, for  $x > 5$ ,  $h''(x) > 0$  and  $h'(x) < 0$ , then [curve  $h$ ] is concave up and down, here [at the interval (3) , 5]] descends until 5, concave upwards. If the limit from  $h(x)$  for  $x$  to infinity is equal to -2, it means that the graph [after 5] cannot touch this axis [line  $y = -2$ , as an asymptote flat]

P: There is a condition  $h'(-2) = h'(3) = 0$ , what does that mean?

L1.13:  $h'(-2) = h'(3) = 0$  means the slope is 0, or the tangent at that point is parallel to the x-axis, at  $x = -2$  inflection points, at  $x = 3$  turning points. Oh yeah suppose at point  $x = -4$  the fracture point. [L shows a graph sketch of the function  $h$  corresponding to L 1.13.]

### [Analysis:

A: Object

As previously described, L can consistently encapsulate processes of adjacent or overlapping intervals at  $x \geq -2$  into an object. This object is also part of the mature scheme in trailer 1, which is also associated with the object in trailer 2, see L1.08. The encapsulated processes are descriptions of intervals of  $-2 \leq x < 0$ ,  $x = 0$ ,  $0 < x < 3$ ,  $3 < x < 5$ , and  $x > 5$ , see L1.07- L 1.13.

B: Object

In Snippet 3, L also encapsulates the processes of all the properties of the  $h$  function at all intervals in the  $h$  domain part, namely  $x \geq -2$ , so that an object is formed. The object formed is part of a mature schema in Snippet 1. Assuming the encapsulated processes are the inflection point descriptions in  $x = -2$  the description  $h'(x)$  and  $h''(x)$  for  $-2 < x < 0$ , the description of the limit of  $h'(x)$  for  $x$  approaches 0 is infinity around  $x = 0$ , the description of  $h'(x)$  and  $h''(x)$  The chest interval  $0 < x < 3$ , the description of the maximum equinox at  $x = 3$ , description of  $h'(x)$  and  $h''(x)$  for the interval  $3 < x < 5$ , description of  $h'(x)$ ,  $h''(x)$  and the limit of  $h(x)$  for  $x$  to infinity at  $x > 5$ , see L 1.07- W 1.13.]



Based on the snippets of genetic decomposition, the following description is generated. The interesting thing about the results of the task-based interview with L is that L can sketch the graph of the  $h$  function accurately. L coordinates each  $h$  domain interval with all the properties of the given function (see Figure 1). For intervals, L can coordinate the first derivative, the second derivative and the limit of infinity and  $h(x)$  (see L 1.04). In the interval  $-4 < x < -2$ , L can coordinate the first derivative, and the second derivative (see L 1.03), from the coordination of two intervals and the properties contained therein, L describes the point at  $x = -4$  as a fracture point (cusp) (See L1.06). The property at point  $x = -2$ , L coordinates it with the properties of the function that exist in two adjacent intervals, namely the interval  $-4 < x < -2$  and  $-2 < x < 0$ , so he concludes it as an inflection point (See L 1.08). Around  $x = 0$ , L coordinates all properties that are in adjacent intervals, namely the first derivative, second derivative, and the limit of the first derivative, which then L concludes that the graph is steep, almost close to the  $y$ -axis (see L1.07). For intervals  $0 < x < 3$ , L coordinates the first derivative  $ma$  and the second derivative, and at  $x = 3$ , is described as the maximum turning point as a result of coordination between  $h'(3) = 0$ . with the properties that exist at intervals adjacent to  $x = 3$  (see L 1.10). At  $3 < x < 5$ , L coordinates the first derivative and the second derivative, and at intervals  $x > 5$ , L coordinates the first derivative, the second and infinite limit.

From the interesting points above, the following conclusions can be made. L Can sketch the graph of the  $h$  function accurately, by coordinating all the properties of the given function in the  $h$  domain. The description of genetic decomposition analysis about the condition of the function trait and the function domain  $h$ , can describe the cognitive processes of the students. For the conditions of the function domain interval  $h$ , L can coordinate the object object about intervals that are adjacent or overlap in the  $h$  domain so that a coherent interval scheme is formed, see Snippet 1 - Snippet 3. Based on the theoretical characteristics of the triad, for the given interval conditions, L entered the trance level. For the conditions of the nature of the function  $h$ , L can coordinate objects about the inherent properties of the function, so that a coherent function characteristic scheme is formed, see Snippet 1 - Snippet 3. L's thought process while solving the  $h$  function graph problem is as follows. L can coordinate the process-object of all the properties of a given function, with intervals that are close together or overlapping the entire  $h$  domain to form a mature scheme of sketches of the  $h$  function graph. These results provide reinforcement of relevant research results, such as (Baker, Cooley, & Trigueros, 2000), that trans level students can demonstrate a coherent scheme in describing graphs based on overlap and connected or unconnected intervals. Another finding that the cognitive process of trans level students is that they can build an object about infinite lines parallel to a certain line. Encapsulation activity produces correct understanding based on the properties of a concept (Herawaty, Khrisnawati, Widada, & Mundana, 2020)(Widada & Herawaty, 2018)(Widada, Herawaty, Nugroho, & Anggoro, 2019a). Therefore, students who are at the trance level are able to solve math problems correctly. Shiva will be more meaningful in solving problems, if taught with learning that is close to the minds of students, close to local culture (Andriani et al., 2020) (Widada, Herawaty, Ma'rifah, & Yunita, 2019) and take advantage of virtual learning (like youtube) (Nugroho, Widada, & Herawaty, 2019).

The description of trans level students provides an overview of the cognitive processes of students in understanding mathematics, especially calculus. With a more complete description this can be a further study, namely developing the characteristics of each level of student cognitive processes.

### CONCLUSION

The research subject (L) is at level 4, namely the trans level. L can coordinate the process-object of all the properties of a given function, with adjacent or overlapping intervals in all  $h$  domains so that a mature scheme of function graph sketches is formed. The cognitive process produces an accurate graph sketch of the function.

## REFERENCES

- A, R., Masriyah, M., & Manuharawati, M. (2019). Difficulties of Undergraduate Students to Understand of 2nd Calculus. *International Journal of Trends in Mathematics Education Research*, 2(1), 26. <https://doi.org/10.33122/ijtmer.v2i1.35>
- Andriani, D., Widada, W., Herawaty, D., Ardy, H., Nugroho, K. U. Z., Ma'rifah, N., ... Anggoro, A. F. D. (2020). Understanding the number concepts through learning Connected Mathematics (CM): A local cultural approach. *Universal Journal of Educational Research*, 8(3), 1055–1061. <https://doi.org/10.13189/ujer.2020.080340>
- Baker, B., Cooley, L., & Trigueros, M. (2000). A Calculus Graphing Schema. *Journal for Research in Mathematical Education*, 31(5).
- Baker, B., Cooley, L., Trigueros, M., & Trigueros, M. (2000). A Calculus Graphing Schema. *Journal for Research in Mathematics Education*, 31(5), 557–578. <https://doi.org/10.2307/749887>
- Dorier, J.-L. (2002). Teaching linear algebra at university. *ICM, III*(1–3). Retrieved from <http://arxiv.org/abs/math/0305018>
- Dubinsky, E. (2000). Using a Theory of Learning in College Mathematics Courses. *MSOR Connections*, 1(2), 10–15. <https://doi.org/10.11120/msor.2001.01020010>
- Dubinsky, E. (2010). Using a Theory of Learning in College Mathematics Courses. *MSOR Connections*, 1(2), 10–15. <https://doi.org/10.11120/msor.2001.01020010>
- Dubinsky, E., Dautermann, J., Leron, U., & Zazkis, R. (1994). On learning fundamental concepts of group theory. *Educational Studies in Mathematics*, 2–38.
- Dubinsky, E., & McDonald, M. A. (2000). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. USA: Georgia State University.
- Fatimah, S., & Yerizon. (2019). Analysis of difficulty learning calculus subject for mathematical education students. *International Journal of Scientific and Technology Research*, 8(3), 80–84.
- Herawaty, D., Khrisnawati, D., Widada, W., & Mun anda, P. (2020). The cognitive process of students in understanding the parallels axiom through ethnomathematics learning. *IOP Conf. Series: Journal of Physics: Conf. Series 1470 (2020) 012077* Doi:10.1088/1742-6596/1470/1/012077, 1470, 1–8. <https://doi.org/10.1088/1742-6596/1470/1/012077>
- Nugroho, K. U. Z., Widada, W., & Herawaty, D. (2019). The Ability To Solve Mathematical Problems Through Youtube Based Ethnomathematics Learning. *International Journal of Scientific & Technology Research*, 8(10), 1232–1237.
- Raj Acharya, B. (2017). Factors Affecting Difficulties in Learning Mathematics by Mathematics Learners. *International Journal of Elementary Education*, 6(2), 8. <https://doi.org/10.11648/j.ijeeedu.20170602.11>
- Tall, D. (1993). Students' Difficulties in Calculus. *Proceedings of Working Group 3 on Students' Difficulties in Calculus, ICME-7*, (Proceedings of Working Group 3 on Students' Difficulties in Calculus, ICME-7), 13–28. <https://doi.org/Canada>
- Tarmizi, R. A. (2010). Visualizing students' difficulties in learning calculus. *Procedia - Social and Behavioral Sciences*, 8(February), 377–383. <https://doi.org/10.1016/j.sbspro.2010.12.053>
- Tsamir, P., & Dreyfus, T. (2002). Comparing Infinite Sets - a process of abstraction. The Case of Ben. *The Journal of Mathematical Behavior*, 21(1).
- Widada, W. (2015). THE EXISTENCE OF STUDENTS IN TRANS EXTENDED COGNITIVE DEVELOPMENT ON LEARNING OF GRAPH THEORY. *Jurnal Math Educator Nusantara*, 1(1), 1–20.
- Widada, W. (2016). Profile Of Cognitive Structure Of Students In Understanding The Concept Of

- Real Analysis. *Journal of Mathematics Education (Infinity)*, 5(2), 83–98.  
<https://doi.org/10.22460/infinity.v5i2.215>
- Widada, W. (2017). Beberapa Dekomposisi Genetik Siswa dalam Memahami Matematika. *Jurnal Pendidikan Matematika Raflesia*, 1(1), 44–54.
- Widada, W., & Herawaty, D. (2017). Dekomposisi Genetik tentang Hambatan Mahasiswa dalam Menerapkan Sifat-sifat Turunan. *Jurnal Didaktik Matematika*, 4(2), 136–151.  
<https://doi.org/10.24815/jdm.v4i2.9216>
- Widada, W., & Herawaty, D. (2018). The Effects of the Extended Triad Model and Cognitive Style on the Abilities of Mathematical Representation and Proving of Theorem. *Advances in Social Science, Education and Humanities Research*, 218(ICoMSE 2017), 89–95.  
<https://doi.org/10.2991/icomse-17.2018.16>
- Widada, W., Herawaty, D., Ma'rifah, N., & Yunita, D. (2019). Characteristics of Students Thinking in Understanding Geometry in Learning Ethnomathematics. *International Journal of Scientific & Technology Research*, 8(11), 3496–3503.
- Widada, W., Herawaty, D., Nugroho, K. U. Z., & Anggoro, A. F. D. (2019a). The Scheme Characteristics for Students at the Level of Trans in Understanding Mathematics during Etno- Mathematics Learning. *Advances in Social Science, Education and Humanities Research*, 253(Aes 2018), 417–421.
- Widada, W., Herawaty, D., Nugroho, K. U. Z., & Anggoro, A. F. D. (2019b). The Scheme Characteristics for Students at the Level of Trans in Understanding Mathematics During Etno-Mathematics Learning. *3rd Asian Education Symposium*, 8(Aes 2018).  
<https://doi.org/10.2991/aes-18.2019.95>
- Widada, W., Nugroho, K. U. Z., & Sari, W. (2018). The Ability of Mathematical Representation through Realistic Mathematics Learning Based on Ethnomathematics Teori APOS sebagai suatu alat analisis dekomposisi genetik terhadap perkembangan konsep matematika seseorang The Scheme Characteristics for Student. *Semin. Adv. Math. Sci. Eng. Elem. Sch. Mercur. Hotel Yogyakarta*, 16.