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Stability Analysis of Predator-prey Model with Stage-structured and the Effect of Harvesting

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Abstract: Predator-prey model is the study to show the interaction between prey and preda- tor. A stage-structured model is introduced where the mature and immature of the species is considered. Also, the effect of harvesting is taken into account for sustainable development. The model in this paper is explained by using the ordinary differential equations to show the dynamic behavior of the predator and prey. Hence, the predator-prey model with stage- structured in prey with the effect of harvesting in predator is considered in this paper. In analyzing the model, the stability of the equilibrium point is obtained and described by using the properties of eigenvalues and Routh-Hurwitz criteria. Finally, numerical simulations are given to verify the analytical results with the help of graphical illustrations.

Kata Kunci: Harvesting Effect; Predator-pre;, Routh-Hurwitz Criteria; Stability Analysis; Stage-structured

INTRODUCTION

The interaction between predator and prey is explained by Lotka and Volterra. Since 1920, Lotka and Volterra have given an explanation of the general dynamic system on the interaction between predator and prey by using autonomous ordinary differential equation. From their study, the research on predator-prey model has attracted the interest among mathematicians and ecologists, particularly in the ecology field [1]. However, Lotka-Volterra model is described as a general system. In the real world, the stage-structured model, featuring both immature and mature species, should be considered. Hence, to study the behaviors of the predation model, stage-structured models have received great attention in recent years [2]. Predator-prey interaction is the interaction in consumer-resource that served as a conceptual foundation in understanding the influences of predation on ecology and evolutionary biology. The predator-prey interaction is concern with the transfer of energy and nutrients between species such that one species is consumer while the other is a resource [3]. Also, the modeling of ecosystems requires learning and understanding the tools that effect the development of species based on their existence and stability [4–6]. In the study conducted in [7], they used modeling approach to species and its ecological function with informative descriptions of life on earth.

In a classical population ecology, the aim of predator-prey interactions in terms of two components is the act of finding, capturing, and consuming prey. This relationship contains a functional response that controls the outcome of the interaction with predator. However, predator-prey interactions in reality are dynamic. Prey will respond to their predator, which may reduce predator hunting success. In return, predators will respond to prey adaptive tactics in order to increase their hunting success. Hence, such behaviors influenced the development of hunter and victim process. Liu et al in [8] proposed a recent progress on stage-structured population dynamics and focused on the single species model with stage-structure and assumes only that the mature species can reproduce, in which the birth rate of mature population depends on the population density. Also, Al-Omari in [9] mentioned that many population species go through two or more life stages as they proceed from birth to death. Most of the pevious models in the literature assumed that single species have similar capabilities to hunt or reproduce. However, most species consist of at least two stages, immature and mature. There fore, it is practical to introduce the stage-structure into the predator-prey models.

The dynamics of stage-structure predator-prey system with Monod-Haldane type response function is carried out by Khajanchi in [10]. The dynamic behavior of this system is investigated from the view point of stability and bifurcation. They investigated the change in dynamical behavior of juvenile and adult predators with the interaction of prey species when there is a transition from prey to juvenile predators, c and the conversion rate of immature predators to mature predators, a. This study shows that the positive equilibrium point has multiple stability analysis as the parameters c and a vary. Meanwhile, Lu et al in [11] considered on the stage-structured predator-prey model with predation over juvenile prey revealed that sufficiently large magnitude of interference among predators may lead to extinction of the predator. This study also supports that the pop- ulation is asymptotically stable as the maturation time of prey is between zero to nine months. However, as maturation time of prey increases from nine to 15 months, the systems becomes more unstable. On the other hand, if the maturation time of predator is less than two months, the populations are asymptotically stable and as maturation time of predator is more than two months, populations becomes increasingly unstable. Hence, this paper shows that if maturation time of prey increases, the juvenile predator may lose its stability. Biologically, this means that the shorter immature prey maturation period is helpful to stabilize the system.

Many researchers have discussed the predator-prey models including the effect of harvesting [12–14]. The types of harvesting that has been used by researchers is discussed in [15]. Harvesting in predator-prey interaction, can involve many parts of research such as economists, ecologists and natural resource managers [16]. Also, harvesting has a tough effect on the dynamic evaluation of a population. With no harvesting, a population can be free of extinction rate; however harvesting opens the posibility of incorporation of a positive extinction probability and therefore the potential of extinction in a finite time. On the other hand, if the population is exposed to a positive extinction rate, harvesting can reduce population density to a seriously low level at which extinction becomes real [17].

Regarding fishery in [18, 19], the authors stated that, while it is essential to harvest from the fishery, harvesting must be structured in such a way that the fishermen limit themselves to harvest immature fish since the immature fish have little commercial value compared to mature fish. In addition to serving the commercial purpose of the fishery, the effort of harvesting is also helpful in the protection of the fishery. This can be done by correcting the mesh size of the net so that when nets are located in water, they can catch all the fish except those small enough to swim through the mesh. The general form of the harvest is generally using the expression on the hypothesis of catch-per-unit-effort. This statement referring to catch per unit effort is relational to the stock level [20].

Hence, in this paper, the stage-structured predator-prey model with the effect of harvesting is considered. The behaviors of the model is investigated. Numerical simulation is provided by giving different sets of parameters while the graphical illustration is given for a better understanding of realistic features of the system.

MODEL FORMULATION

In this study, the model is referred to the two types of population which are predator and prey with stage-structured in prey. In the model, the stage-structured of prey (immature and mature) are described as x1 and x2 respectively and predator written as y. The assumptions of the model is as the following

(i) The predator only consumes the immature prey.

(ii) The increasing of density in immature prey is proportional to the birth rate of existing mature prey with a proportionality constant α (written as αx_2); the death rate of immature prey is proportional to the existing immature prey with a proportionality constant r_1 (written as r_1x_1); the maturation rate of immature prey is proportional to the existing immature prey with a proportionality constant γ (written as γx_1); the competition rate among immature prey is proportional to the existing immature prey with a proportionality constant μ_1 (written as $\mu_1 x^2$); the predation rate of immature prey is proportional to the existing immature prey and predator with a proportionality constant β (written as $\beta x_1 y$).

(iii) The density of mature prey is proportional to the maturation rate of existing immature prey with a proportionality constant γ (written as γx_1); the death rate of mature prey is proportional to the existing mature prey with a proportionality constant r_2(written as r_2x_2).

(iv) The density of predator is proportional to the predation rate of existing immature prey and predator with a proportionality constant $k\beta$ (written as $k\beta x_1 y$); the competition rate among predator is proportional to the existing predator with a proportionality constant μ (written as μy^2); the death rate of predator is proportional to the existing predator with a proportionality constant r (written as ry).

According to assumptions in (i),(ii),(iii) and (iv), the predator-prey model with stage-structured in prey can be governed by the following system of differential equations:

$$\frac{dx_1}{dt} = \alpha x_2 - r_1 x_1 - \gamma x_1 - \mu_1 x_1^2 - \beta x_1 y$$
$$\frac{dx_2}{dt} = \gamma x_1 - r_2 x_2 \qquad (1)$$

where α, r, r1, r2, γ, β, μ, μ1 are positive constants.

DIMENSIONLESS MODEL

Model (1) is dimensionless to reduce the parameter used. First, let

$$X_1 = \frac{k\beta}{r_2} x_1, \quad X_2 = \frac{k\beta}{\gamma} x_2, \quad Y = \frac{\mu}{r^2} y, \quad \tau = r_2 t.$$

Then, model (1) is turned to:

$$\frac{dX_1}{d\tau} = aX_2 - bX_1 - cX_1^2 - dX_1 Y$$
$$\frac{dX_2}{d\tau} = X_1 - X_2 \qquad (2)$$
$$\frac{dY}{d\tau} = Y(X_1 - Y - e)$$

where

αγ r₁+γ μ₁ β r

EFFECT OF HARVESTING

Model (2) is extended by including the effect of harvesting. The effect of harvesting is defined as the hypothesis of catch-per-unit effort such that

$$h = qE$$

where

q = catch ability coefficient, E = harvesting effort.

In this study, only the predator is being harvested.

By including the effect of harvesting to the predator population, model (2) is written as:

$$\begin{aligned} \frac{dX_1}{d\tau} &= aX_2 - bX_1 - cX_1^2 - dX_1Y \\ \frac{dX_2}{d\tau} &= X_1 - X_2 \\ \frac{dY}{d\tau} &= Y(X_1 - Y - e) - qEY. \end{aligned} \qquad (3)$$

The possible non-negative equilibrium points for model (3) are:

$$E_0 = (0, 0, 0),$$
$$E_1 = \left(\frac{a-b}{c}, \frac{a-b}{c}, 0\right),$$
$$E_2 = (X_1^*, X_2^*, Y^*),$$

where

$$X_1^* = \frac{a-b+d(e+qE)}{c+d}, \quad X_2^* = \frac{a-b+d(e+qE)}{c+d}, \quad Y^* = \frac{a-b-c(e+qE)}{c+d}.$$

The condition for the existence of equilibrium points are:

(v) for second equilibrium point, E_1: $a > b$.
(vi) for third equilibrium point, E_2: $\frac{b-a}{d} < e + qE < \frac{a-b}{c}$.

The stability of the equilibrium points can be summarized as below:

(i) for first equilibrium point, E_0: $a < b$.

(ii) for second equilibrium point, E_1: $b < a < b + c(e + qE)$.

(iii) for third equilibrium point, E_2: $P(\lambda) = \lambda^3 + A_1\lambda^2 + A_2\lambda + A_3$
where

$$A_1 = \frac{cde + cdqE - ce - cqE + 2ac + ad - bc + a - b + c + d}{c + d},$$

$$A_2 = \frac{-cdq^2E^2}{c + d} + \frac{qE(-2cde - 2ac + ad + bc - bd + cd\, e)}{c + d} + \frac{-cde^2}{c + d} + \frac{e(-2ac + ad + bc - bd + cd - c)}{c + d} + \frac{2a^2 - 3ab + ac + b^2 - bc + a - b}{c + d},$$

$$A_3 = \frac{-cdq^2E^2}{c + d} + \frac{qE(-2cde - ac + ad + bc - bd)}{c + d} + \frac{-cde^2}{c + d} + \frac{e(-ac + ad + bc - bd)}{c + d} + \frac{(a - b)^2}{c + d}.$$

The stability of E_0 and E_1 are obtained by using the properties of eigenvalues. From the prop- erties of the eigenvalue, the equilibrium point is only stable if all the real parts of the eigenvalues are negative. Also, the stability of E_2 is analyzed by using Routh-Hurwitz criteria. Therefore, E_2 is stable if and only if all the conditions $A_1 > 0$, $A_3 > 0$ and $A_1A_2 > A_3$ are fulfilled.

Numerical Simulations

In this section, some values are set to model (3). The numerical simulation is conducted to verify the existence and stability of the equilibrium points. The stability of model (3) is investigated by using the graphical illustrations on stability region for E_1 and E_2 if the parameters a, b, c and e are fixed. In clearer view, harvesting rate (qE) versus predation rate (d) is plotted as in Figure 1.

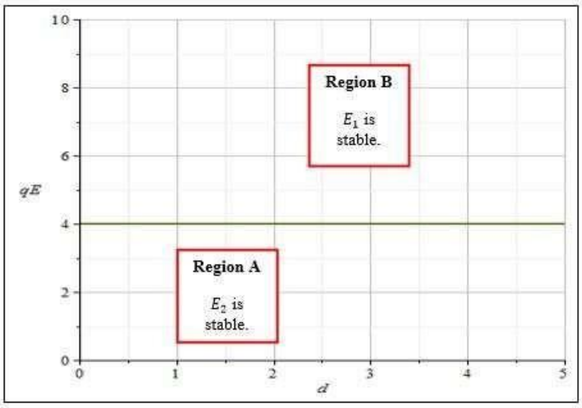

Figure 1: Stability region for E_1 and E_2 if a = 10, b = 5, c = 1 and e = 1

Figure 1 indicates the stability region for the equilibrium points E_1 and E_2. In Region A, E_2 (the coexistence equilibrium point) is stable. In this region, the harvesting rate is low. E_1 is stable in Region B when the rate of harvesting towards the predator is increased.

Table 1: The Stability of the equilibrium points (EP) for different sets of parameter

Parameter	EP	Eigenvalues	Routh-Hurwitz criteria	Stability of EP
d = 3, qE = 0	$E_0 = (0, 0, 0)$	$\lambda_1 = -1$ $\lambda_2 = 0.74$ $\lambda_3 = -6.74$	—	Unstable
	$E_1 = (5, 5, 0)$	$\lambda_1 = 4$ $\lambda_2 = -0.32$ $\lambda_3 = -15.68$	—	Unstable
	$E_2 = (2, 2, 1)$	—	$A_1 = 14$ $A_2 = 21$ $A_3 = 8$ $A_1A_2 = 294$	Stable
d = 3, qE = 1	$E_0 = (0, 0, 0)$	$\lambda_1 = -2$ $\lambda_2 = -6.74$ $\lambda_3 = 0.74$	—	Unstable
	$E_1 = (5, 5, 0)$	$\lambda_1 = -15.68$ $\lambda_2 = -0.32$ $\lambda_3 = 3$	—	Unstable
	$E_2 = (2.75, 2.75, 0.75)$	—	$A_1 = 14.5$ $A_2 = 19.25$ $A_3 = 8.25$ $A_1A_2 = 279.13$	Stable
d = 3, qE = 8	$E_0 = (0, 0, 0)$	$\lambda_1 = -6.74$ $\lambda_2 = -9$ $\lambda_3 = 0.74$	—	Unstable
	$E_1 = (5, 5, 0)$	$\lambda_1 = -15.68$ $\lambda_2 = -0.32$ $\lambda_3 = -4$	—	Stable
	$E_2 = (8, 8, -1)$	Not biologically relevant		

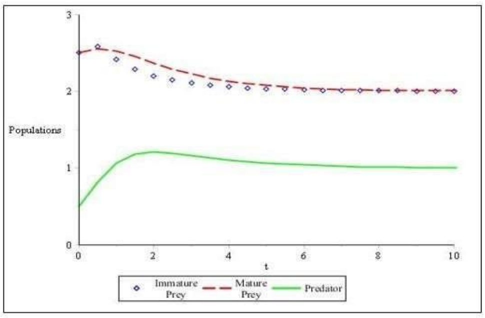

Figure 2: Absence of harvesting

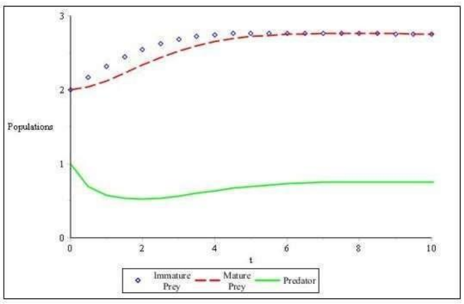

Figure 3: With harvesting (qE=1)

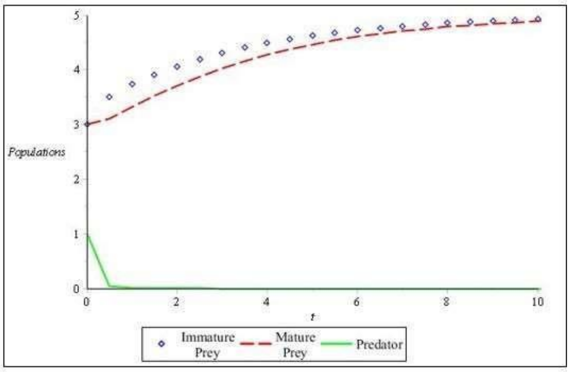

Figure 4: With harvesting (qE=8)

Table 1 is a summary of the stability of equilibrium points for different sets of parameters. From Table 1, three cases are considered which are the absence of harvesting, presence of low harvesting and a higher rate of harvesting. The stability of E_0 and E_1 are obtained by using the properties of the eigenvalues and for E_2, the Routh-Hurwitz criterias are used. The equilibrium point is stable if all the real parts of the eigenvalues are negative. In Routh-Hurwitz criteria, the equilibrium point is stable if the conditions $A_1 > 0$, $A_3 > 0$ and $A_1A_2 > A_3$ are satisfied. In model (3), harvesting is applied to the predator population. As shown in Figure 2, with the absence of harvesting, all populations exist. After applying a slight increase in the rate of harvesting to the predator population, predator population is decreased in density as in Figure 3. Meanwhile, with the higher rate of harvesting, eventually, the predator population will tend to extinction as in Figure 4. Also, Figure 4 shows that, the population of mature and immature prey increase in density since they are free from being consumed by the predator. From the analysis, harvesting gives negative impact on the populations since the harvest population will show a decrease in density and after some time, they tend to extinction.

CONCLUSION

This study described the predator-prey model with stage-structured in prey. The effect of harvesting is also included in the predator population. To examine the behavior of the systems, the existence and stability of the equilibrium points are discussed by using the properties of eigenvalues and Routh-Hurwitz criteria. From the stability analysis of the model, it can conclude that har-vesting gives a negative impact on the population. This is referring to the absence of harvesting, all the populations exist and stable after some time. However, with a slight increase in the rate of harvesting, the harvest population which is predator decreases in density. Also, if a higher rate of harvesting is applied to the predator population, eventually the harvested population will tend to extinction.

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